II. *Solution by Irving Kaplansky, Harvard University*

This is a special case of the problem treated in my note *On a generalization of the "Problème des Rencontres"* [1939, 159]. Putting $n=13$, $a_i=4$, $p_1=\cdots=p_{10}=1$, $p_{11}=p_{12}=p_{13}=0$ in the formula obtained there, we get

$$E^{12}(E^4-4E^3)^{10}0! = E^{42}(E-4)^{10}0! = \sum_{i=0}^{10}(-4)^i\binom{10}{i}(52-i)!$$

for the number of arrangements.

III. *Experimental check by D. H. Browne, Buffalo, N. Y.*

By a trial of several hundred deals, using a single close ruff and a lift cut between each deal and the next, I get an average of 54.5%.

Also solved by E. P. Starke and the proposer.

ADVANCED PROBLEMS

Send all communications about Advanced Problems and Solutions to Otto Dunkel, Washington University, St. Louis, Mo. All manuscripts should be typewritten, with double spacing and with margins at least one inch wide.

Problems containing results believed to be new or extensions of old results are especially sought. The editorial work would be greatly facilitated, if, on sending in problems, proposers would also enclose any solutions or information that will assist the editors in checking the statements. In general, problems in well known text-books or results found in readily accessible sources will not be proposed as problems for solution in this department. In so far as possible, however, the editors will be glad to assist members of the Association in the solution of such problems.

PROBLEMS FOR SOLUTION

4036. *Proposed by L. A. Santaló, Rosario, Argentina*

Let C_1 be an oval with a continuously varying radius of curvature R; at each point of C_1 a normal of length R is drawn exteriorly giving points of a second curve C_2 (which may not be convex); and let A be the area enclosed between the two curves. From a chosen fixed point a vector is drawn parallel to the normal at a point of C_1 and of length R for that point, thus giving as the point varies on C_1 a curve C_3 having the area A_3 and length L_3. If L_2 is the length of C_2 and A_1 is the area of C_1, show that

$$\text{(a)}\quad A = 3A_3; \qquad \text{(b)}\quad L_2L_3 \geqq 8\pi A_1;$$

the equality in (b) is true only when C_1 is a circle.

4037. *Proposed by Cezar Coşniţă, Focşani, Roumania.*

Integrate

$$(x^{n+1}+y^n)y' - x^n y = 0;$$

calculate and examine the radius of curvature of the integral curves at the origin.

4038. *Proposed by V. Thébault, San Sebastián, Spain*

The point M is chosen arbitrarily on a bisector of angle A of the triangle ABC, and let M' be its isogonal conjugate with respect to ABC. Show that the two circles each through M and M' and tangent to the side BC are tangent also to the circumcircle of ABC.

4039. *Proposed by N. A. Court, University of Oklahoma*

The circumcenter of a tetrahedron (T) and any point M are isogonal conjugates with respect to the tetrahedron formed by the centers of the four spheres passing through M and the circumcircles of the faces of (T).

Correction of editorial errors in 3994 [1941, 273]. *Proposed by C. E. Springer, University of Oklahoma*

If

$$a_{11} = a_{22} = a_{33} = \sum_j \binom{K}{j}(n-1)^{K-j}, \qquad j \equiv 0 \pmod 3;$$

$$a_{12} = a_{23} = a_{31} = \sum_j \binom{K}{j}(n-1)^{K-j}, \qquad j \equiv 1 \pmod 3;$$

$$a_{13} = a_{32} = a_{21} = \sum_j \binom{K}{j}(n-1)^{K-j}, \qquad j \equiv 2 \pmod 3;$$

show that the determinant

$$|a_{ij}| = [(n-1)^3 + 1]^K.$$

SOLUTIONS

Symmetric Functions

3980 [1941, 69]. *Proposed by Esther Szekeres, Budapest, Hungary*

The symmetric polynomials $y_1, y_2, \cdots, y_n$ in the variables $x_1, x_2, \cdots, x_n$ are of the degrees indicated by the subscripts, and are algebraically independent. If $f(x_1, x_2, \cdots, x_n)$ is any given polynomial symmetric in the x's, show that it can be expressed as a polynomial in the y's.

Solution by the Proposer

Since the symmetric polynomial $f(x_1, x_2, \cdots, x_n)$ can be expressed as a polynomial in terms of the elementary symmetric functions $\sigma_1, \sigma_2, \cdots, \sigma_n$, it suffices to show that each σ_i can be expressed as a polynomial in the y_i's.

We may write

$$y_k = c_k\sigma_k + g_k(\sigma_1, \sigma_2, \cdots, \sigma_{k-1}),$$

where the second term in the right member is a polynomial in the indicated arguments whose terms are of weights not exceeding k, or in other words it is the sum of terms such as