Solution by J. B. Kelly, Langley Field, Va. Set $x=p+p^{-1}$, and let P_n denote the n-rowed determinant. Then

$$P_1 = x = p + p^{-1}, \qquad P_2 = x^2 - 1 = p^2 + 1 + p^{-2}.$$

Since $P_n = xP_{n-1} - P_{n-2}$ and $p^{n+1} - p^{-n-1} = (p+p^{-1})(p^n - p^{-n}) - (p^{n-1} - p^{-n+1})$, we can prove by induction that

$$P_n = p^n + p^{n-2} + \cdots + p^{-n+2} + p^{-n} = (p^{n+1} - p^{-n-1})/(p - p^{-1}).$$

Set $p = e^{\theta i}$, so that $x = 2\cos\theta$. Then

$$P_n = \frac{\sin (n+1)\theta}{\sin \theta},$$

which vanishes when

$$\theta = \frac{\pi}{n+1}\cdot$$

Also solved by J. H. Butchart, Paul Civin, A. B. Farnell, N. J. Fine, Marguerite Z. and E. A. Hedberg, G. A. Hedlund, Irving Kaplansky, O. E. Lancaster, Leonard McFadden, M. F. Smiley, Alan Wayne, and the proposers.

Editorial Note. The following alternative solution is less significant, but rather entertaining. If $x = 2\cos\theta$, we have (identically)

$$\begin{aligned}
x \sin\theta - \sin 2\theta &= 0,\\
\sin\theta - x\sin 2\theta + \sin 3\theta &= 0,\\
- \sin 2\theta + x \sin 3\theta - \sin 4\theta &= 0,\\
\cdots\cdots\cdots\cdots &\\
\pm \sin (n-1)\theta \mp x \sin n\theta \pm \sin (n+1)\theta &= 0.
\end{aligned}$$

If $\theta = \pi/(n+1)$, the last term of the last equation vanishes, so we can formally eliminate $\sin\theta : \sin 2\theta : \cdots : \sin n\theta$ from the n equations (with that term omitted), obtaining the desired result immediately.

Hedlund remarks that

$$P_n = x^n - \binom{n-1}{1} x^{n-2} + \binom{n-2}{2} x^{n-4} - \cdots .$$

In other words, $P_n = U_n(x/2)$, where U_n is the Tschebyscheff polynomial of the second kind. (*Cf.* E 620 [1945, 44–46] and E 629 [1945, 100–101].) The determinantal expansion for $\sin (n+1)\theta$ was first given by Studnička in 1897.

An Almost Linear Set of Segments

E 649 [1944, 586]. *Proposed by L. A. Santaló, Rosario, Argentine Republic*

A set of parallel line segments will be called "linear" if all of them can be cut by one straight line. Show by an example that an infinite set of parallel segments in one plane may have the property that every subset of three is linear while the whole set is not linear. (The segments are understood to be "open": not including their end points.)

I. *Solution by N. J. Fine, Lukas-Harold Laboratory, Indianapolis.* Let $f(x)$ be any real, positive-valued function such that $\lim_{|x|\to\infty} f(x)=0$. Then the set of ordinates $\{0<y<f(x)\}$ satisfies the required conditions. As a matter of fact, every finite subset is "linear."

A discrete set having the same property may be obtained by selecting those ordinates for which x is integral.

II. *Solution by Howard Eves, Syracuse University.* The segments

$$a_0,\ a_1,\ a_2,\ \cdots,\ \text{and } b,$$

defined as follows, are seen to satisfy the requirements.

$$a_i:\quad x = 2^{-i},\qquad 0 < y < 1;$$

$$b:\quad x = -1,\qquad 2 < y < 3.$$

Also solved by L. M. Kelly and the proposer. No *finite* set of segments can have this property; nor can any set of *intervals* (or "closed" segments). In other words, if every three of the intervals are linear, then the whole set is. See L. A. Santaló, "Complemento a la nota: sobre conjuntos de paralelepípedos de artistas paralelas," *Publicaciones del Instituto de Matemáticas*, vol. 3, no. 7 (1942); *Mathematical Reviews*, vol. 4 (1943), p. 112.

ADVANCED PROBLEMS

Send all communications about Advanced Problems and Solutions to Otto Dunkel, Washington University, St. Louis 5, Mo. All manuscripts should be typewritten, with double spacing and with margins at least one inch wide.

Problems containing results believed to be new or extensions of old results are especially sought. The editorial work would be greatly facilitated, if, on sending in problems, proposers would also enclose any solutions or information that will assist the editors in checking the statements. In general, problems in well known text-books or results found in readily accessible sources will not be proposed as problems for solution in this department. In so far as possible, however, the editors will be glad to assist members of the Association in the solution of such problems.

PROBLEMS FOR SOLUTION

4161. *Proposed by R. E. Gaines, University of Richmond*

Along a straight road a miles long there are n persons. What is the probability that no two persons are less than b miles apart?

4162. *Proposed by H. S. M. Coxeter, University of Toronto*

Prove, by the methods of real projective geometry, that if a projectivity $P \barwedge P'$ on a conic is not an involution, the envelope of PP' is a conic. (For a complex proof, see H. F. Baker, *Principles of Geometry*, Cambridge 1922 or 1930, p. 52.)